FFT Laboratory Experiment 2

Aliasing

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Purpose:

The goal of this experiment is to demonstrate that aliasing occurs if the effective sampling rate is below the Nyquist rate of the input signal. A sinusoidal input is also used for this experiment. The theoretical basis for this experiment is illustrated in Figure 3. 1.

Procedure:

1. Connect a signal generator to channel 1 of the oscilloscope. Select a 3.5 *V* (peak-to-peak) sinusoidal signal with a fundamental frequency of approximately 10 kHz. Use *Autoscale* to display the time-domain waveform.

2. Next depress the ± key and then select the *On* entry under the Function 2 menu. Next depress the 1 key on the display and then select *Off* under the 1 menu entry. Use the *Time /Div* control to select an effective sampling rate of 50 *kSa* / s. Refer to Figure 3. 1 to select the settings for the FFT menu.

3. Using the signal generator's frequency control, progressively increase the frequency of the sinusoid to roughly 24 kHz, allowing the FFT display to stabilize at several points along the way. You should see the peak lobe of the FFT display move to the right as the sinusoidal frequency is increased.

4. Continue to slowly increase the sinusoid's fundamental frequency. Aliasing occurs as the frequency of the sinusoid exceeds 25 kHz. As the frequency is swept from 25 kHz to 50 kHz, the main-lobe moves to left on the display. As the sinusoid's frequency is swept from 50 kHz to 75 kHz, the main lobe moves to the right on the FFT display.

5. Set the sinusoid frequency to 40 kHz. Use the *Cursors* and *Find Peaks* functions to measure the peak frequency displayed by the FFT. Because of aliasing, the FFT should erroneously indicate that the peak occurs at about 10 kHz.

6. Finally, repeat step 5 only this time change the effective sampling rate to 100 *kSa* / s. Since the effective sampling rate is greater than the Nyquist rate, the spectrum is accurately approximated.

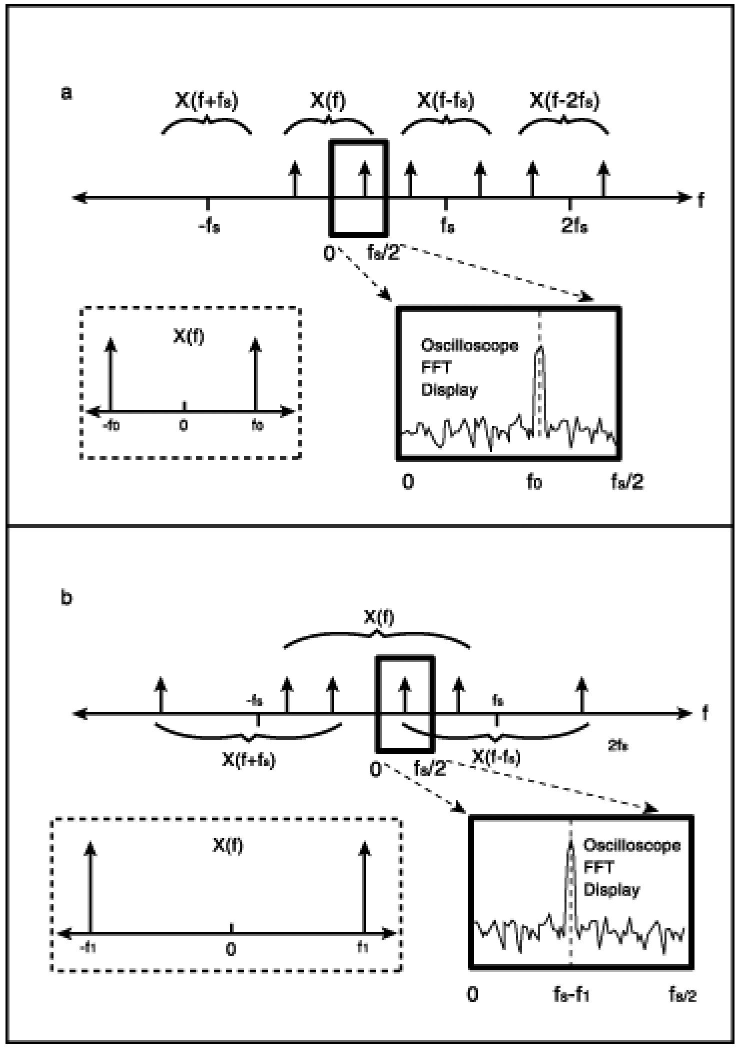
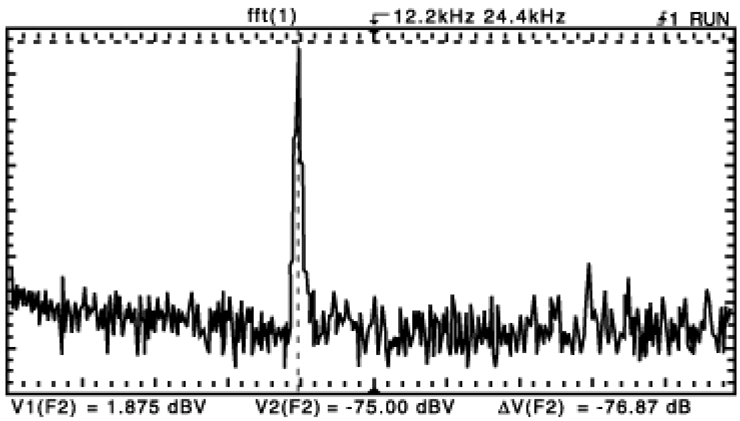


Figure 3.1

a) **In the first portion of the figure, the sinusoid is sampled at a rate which is greater than the Nyquist frequency of the sinusoid. The spectrum of the sinusoid is denoted by *X(f)* and is shown in the "dashed-line" box. Sampling the signal results in a periodic replication of the original spectrum as shown above. The "solid-line" box shown in the figure represents the portion of the spectrum that is displayed by the oscilloscope. Note that the maximum frequency that can be displayed on the oscilloscope is $f_S/2$. Since the individual replicas do not overlap (that is, $f_0 < f_S/2$) the spectrum of the input signal is accurately represented.**

b) **In the second half of the figure, the sinusoid is under-sampled which results in aliasing. When the original spectrum is periodically replicated, the portion of the spectrum due to *X(f)* does not appear within the frequency range of the oscilloscope display. However, the portion of the spectrum due to $X(f-f_S)$ *does* appear in the frequency range of the display. Because of aliasing, the oscilloscope display is not an accurate display of the signal spectrum.**



Function2: Menu		
Effective Sampling Rate	**Units/div**	**Ref Level**
50.00 kSa/s	**10.00 dB**	**5.00 dBV**

FFT Menu		
Center Freq.	**Freq. Span**	**Window**
12.21 kHz	**24.41 kHz**	**Hanning**

Figure 3.2

The 1024 point DFT of a 3.5 *V* (peak-to-peak), 10 kHz sinusoid.

Questions

1. Since the spectral resolution of the FFT is limited by the effective sampling rate, and since we must sample above the Nyquist rate to avoid aliasing, is it possible to improve spectral resolution by zeropadding?

2. If a 120 kHz sinusoid is sampled at 50 *kSa / s,* at what frequency will the aliased 100 kHz component appear in the FFT display?

3. Is aliasing affected by the choice of window functions?

Conclusion:

The frequency range of the FFT display is from 0 to $f_S/2$ Hz. Any frequency components of the input signal which are higher than $f_S/2$ will alias and appear as a lower frequency in the FFT display.

Answers to Questions

1. Zero-padding is a technique that is often used to improve the *appearance* of spectral plots, and for interpolating between frequency samples of a fixed length FFT. Zero-padding is effective in producing more frequency samples of the periodically replicated signal spectrum. For example, the plots shown in parts (b) and (c) of Figure 1.6 were produced using zero padding. Since a true improvement in spectral resolution comes from a longer time-duration window, zero padding is a somewhat artificial method for improving resolution. The most effective way to improve spectral resolution is to increase the time duration of the window function by either taking more samples (increasing N) or by lowering the sampling rate.

2. This signal is clearly under sampled. The Fourier transform component of the original signal, which is at 120 *k*Hz, is periodically replicated every 50 *k*Hz across the entire spectrum. A quick calculation ($120\,k-2(50\,k)=20\,k$) reveals that the 120 *k*Hz component aliases to 20 *k*Hz.

3. No, aliasing is a function of the sampling rate.